Non-parametric Spectrum Sensing based on Censored Observations in Quasi-static Fading Channel for Cognitive radio

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Abstract—**The detection of primary user (PU) is a challenging task for a cognitive radio user to access the spectrum opportunistically. In this paper, a spectrum sensing method based on censored observations is proposed as Censored Anderson Darling sensing (CAD). We evaluate the performance of the CAD sensing method with Monte Carlo simulations. It is shown that the proposed method outperforms the energy detection at about** 6 **dB gain over quasi-static fading channel at lower signal to noise ratio. Also, it gives better detection performance compared to AD sensing and OS based sensing methods which have recently been proposed in the literature.**

Keywords*—*Spectrum sensing, goodness of fit test, quasi-static channel, censored data

I. Introduction

Today's wireless networks are characterized by fixed spectrum assignment policy. With ever increasing demand for frequency spectrum and limited resource availability, the FCC has decided to make a paradigm shift by allowing more and more number of secondary users (SU) to transmit their signals in licensed bands for utilizing the available spectrum of primary users (PU) efficiently. This can be achieved using Cognitive radio (CR) [1]. One of the most important components in Cognitive radio is spectrum sensing. The main function in spectrum sensing is to detect the PU (or licensed user). This task is performed by SU (or unlicensed user) which can use the spectrum of PU such that they do not cause interference to PU. The spectrum sensing function is suffered by multipath fading, receiver's uncertainty, interference etc. Therefore, the design of a spectrum sensing algorithm for future wireless communications is a challenging problem in the research community.

In the last couple of years, many efforts are put by researches to provide spectrum access in an opportunistic way. There are different spectrum sensing techniques proposed under the category of parametric sensing in which some information about PU is available at SU. The different parametric sensing methods are Cyclo-stationary detection, matched filter, waveform-based sensing etc [2] [3]. In category of nonparametric sensing, Energy Detection (ED) [4] and Goodness of Fit (GoF) tests based sensing like Anderson Darling (AD) sensing [5], Kolmogorov-Smirnov (KS) sensing [6], Student t- sensing [7] and Order statistics based sensing [8] are proposed wherein no information about PU is required at SU.

The energy detection (ED) is the most common method for spectrum sensing due to its low complexity. However, the performance of the ED degrades at low signal-to-noise ratio (SNR) with uncertainty in noise power. Also, the detection performance is degraded at the less number of observations. In this scenario of low SNR and received observations, GoF test based sensing is preferred. GoF based sensing always means a statistical test for the presences of certain distribution [9]. More specifically, all received observations are independent and identically distributed (i.i.d) random variables with cumulative distribution function (CDF), denoted by F. In this kind of sensing, the test of the null hypothesis ($F = F_0$) against the alternative hypothesis ($F \neq F_0$) has been done, where F_0 is an available CDF. For performing any GoF test, the empirical CDF (ECDF) is determined from the received observations. This ECDF is compared with the known CDF (F_0) under the null hypothesis. The distance of the ECDF from the CDF decides whether PU is present or absent.

Based on this GoF testing method, AD sensing was proposed, wherein a special weight function has been used to give more emphasis to the tails of the CDF. Furthermore, in [10], AD test is used for the detection of PU under the condition of unknown noise power. The Student's t-distribution is used for the testing of null hypothesis instead of gaussian distribution. Recently in [11], the distance between the ECDF of the received observations and the known CDF, is measured using characteristic functions, instead of the CDFs, has been considered for the testing of null hypothesis. Also, GoF testing based on order statistics in [8], has been proposed for an AWGN channel.

All types of GoF based sensing methods which are proposed in literature so far, have used all observations to determine ECDF. However, the distance of the CDF and ECDF is higher especially at the right tail due to less number of observations. This incomplete information of CDF on the right tail introduces an error in determining statistics in AD sensing, especially at low SNR. To overcome this, we have used the concept of censored data which has already been used in survival analysis [12]. In view of this, we drop some observations in the right tail, which carry incomplete

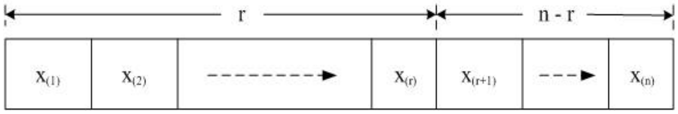

Fig. 1 Number of received (N) and censored $(N-R)$ observations

information for the CDF.

In this paper, we have proposed a nonparametric sensing method using GoF testing based on censored observations, also called Censored Anderson Darling (CAD) sensing. In this method, the observations from right tail are removed and we use modified statistic for the testing of null hypothesis as derived in [13]. This statistic has been obtained by modifying the upper limit of the integration. We have found that CAD sensing outperforms the AD sensing at lower values of SNR and less numbers of received observations under a quasi-static channel. Furthermore, the processing become simpler for the detection of PU. We have also compared CAD sensing with ED and OS based sensing methods.

The rest of the paper is organized as follows. The problem of spectrum sensing as GoF testing for censored observations is formulated as null hypothesis testing problem in Section II. In Section III, the detection performance of the CAD sensing algorithm is presented and compared with OS, AD and ED sensing methods. Finally, the paper is concluded in Section IV.

II. Goodness of fit testing for censored observations

Let $\mathbf{y} = [y_1, y_2, ..., y_N]^T$ be the received signal vector at the secondary user (SU), where N denotes total number of observations. We assume received observations are real valued and each y_i is represented as,

$$y_i = \sqrt{\rho} hs + w_i, \quad i = 1, 2, 3, \cdots\cdots N, \tag{1}$$

where $s \in \{0, 1\}$, ρ is the received SNR, h represents the fading factor, which is assumed to be random variable with the standard normal distribution. We also assume that the channel is quasi-static rayleigh fading. In (1), w_i, where $1 \leq i \leq N$, denotes noise samples. The CDF of w_i is denoted by $F_0(y)$. In (1), $s = 1$ and 0 denote presence and absence of PU respectively. Without loss of generality, we assume that all N observations are in ascending order. It means $y_1 \leq y_2 \leq \cdot\cdot, y_N$. Now, we retain first R observations and drop or censor the last $N-R$ observations as shown in Fig. 1. Hence, y_R is the highest valued observation. This method of censoring $N-R$ highest valued observations is known as right censoring with Type-2 [12].

In this scenario, the problem of spectrum sensing as null hypothesis testing problem as GoF testing is defined as [5],

$$\begin{aligned} H_0 &: F_Y(y) = F_0(y) \\ H_1 &: F_Y(y) \neq F_0(y) \end{aligned} \tag{2}$$

For CAD sensing, we use modified Cramer-von Mises GoF statistic to measure distance between $F_Y(y)$ and $F_0(y)$. Let $F_n(y)$ be the Empirical Cumulative Distribution Function (ECDF) of the received observations $\mathbf{y}$, which can be expressed as

$$F_n(y) = \frac{|\{i : y_i \leq y, 1 \leq i \leq N\}|}{N}, \tag{3}$$

where $|\cdots|$ indicates cardinality. In this case, based on the asymptotic distribution of censored observations, statistic can be expressed as [13],

$${}_{q,p}A_N^2 = N \int_q^p \frac{(F_n(y) - F_0(y))^2}{F_0(y)(1-F_0(y))} dF_0(y), \quad 0 \leq q < p \leq 1,$$

where p denotes censoring ratio which can be expressed as

$$p = \lim_{n \to \infty} \frac{R}{N}.$$

Here, we take $q = 0$. In this case, statistic can be written as,

$${}_pA_N^2 = N \int_0^p \frac{(F_n(y) - F_0(y))^2}{F_0(y)(1-F_0(y))} dF_0(y) \tag{4}$$

The above quadratic statistics ${}_pA_N^2$ can be solved using integration by parts and approximated as [13],

$$\begin{aligned} {}_pA_N^2 = &-\frac{1}{N}\sum_{i=1}^{R}(2i-1)(lnz_i - ln(1-z_i)) - 2\sum_{i=1}^{R} ln(1-z_i) \\ &- \frac{1}{N}[(R-N)^2 ln(1-z_R) - R^2 lnz_R + n^2 z_R], \end{aligned} \tag{5}$$

where $z_i = F_0(y_i)$. For sensing at secondary user, based on censored observations, H_0 is rejected when ${}_pA_N^2 > \lambda$, where λ is the value of threshold. The λ is selected such that the false alarm probability (P_f) under H_0 is at a desired level α,

$$\alpha = \mathbb{P}\{\, {}_pA_N^2 > \lambda | H_0\} \tag{6}$$

To find λ, it is worth to mention that the distribution of ${}_pA_N^2$ under H_0 is independent of the $F_0(y)$. To observe this, apply probability integration transform (PIT) for available observations. Hence,

$${}_pA_N^2 = N \int_0^p \frac{(F_z(z) - z)^2}{z(1-z)} dz, \tag{7}$$

where $z = F_0(y)$ and $F_z(z)$ denotes ECDF of z_i. Here, $z_i = F_0(y_i)$ for $1 \leq i \leq R$. All statistics of observations up to z_R are independent and uniformly distributed over $[0, p]$, where $p \in [0, 1]$. As shown in [5] for AD sensing, the distribution of A_c^2 is independent of the $F_0(y)$. The same is also true for the distribution of ${}_pA_N^2$. As given in [9], the value of λ is determined for a specific value of P_f and censoring ratio p. For example, when $P_f = 0.05$ and $p = 0.4$, the value of λ is 1.133.

Let us summarize, the above discussion in the following steps for CAD sensing algorithm:

Step:1 Find the threshold λ for a given probability of false alarm P_f using (6).

Step:2 Sorting all the observations in ascending order, we get

$$y_1 \leq y_2 \leq \cdot\cdot \leq y_R \leq y_{R+1} \leq \cdot\cdot \leq y_N,$$

where $y_{R+1} \leq y_{R+2} \cdot\cdot \leq y_N$ observations are censored.

Step:3 Calculate the required test statistic ${}_pA_N^2$ for the observations $y_1 \leq y_2 \leq \cdot\cdot \leq y_R$ as defined in (5).

Step:4 If ${}_pA_N^2 > \lambda$, then reject null hypothesis H_0.

Step:5 Compute performance metric as Probability of Detection (P_d) with a given value of P_f.

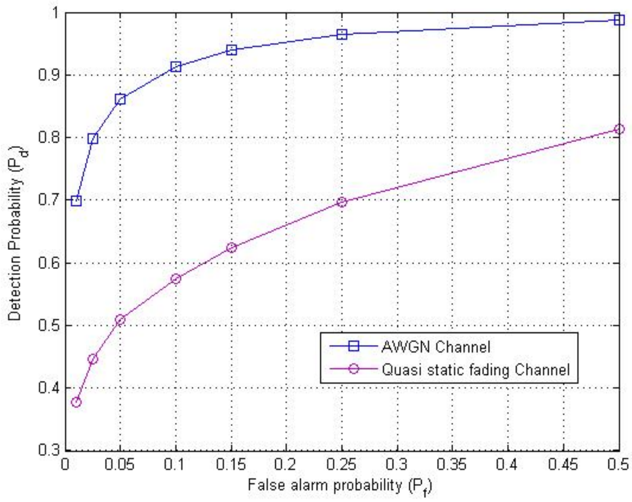

Fig. 2 ROC graph for Censored Anderson-Darling sensing at SNR = -2 dB

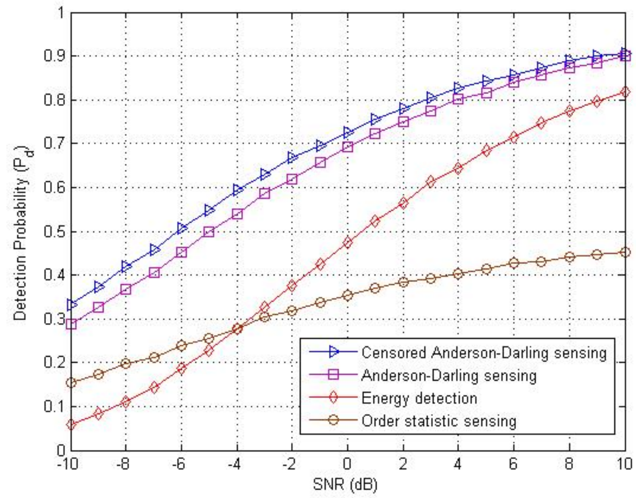

Fig. 4 Probability of detection for CAD, AD, OS and ED sensing at $P_f = 0.05$ under Quasi-static fading channel

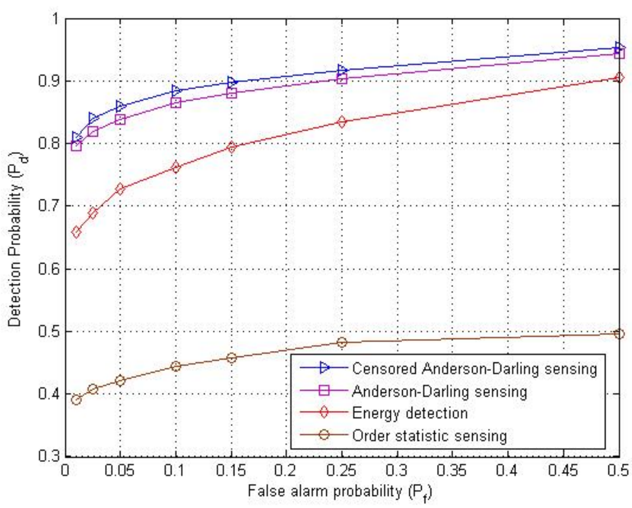

Fig. 3 ROC graphs for CAD, AD and ED Sensing at SNR = 6 dB under Quasi-static fading channel

III. SIMULATION RESULTS

In this section, we have shown the performance of the CAD sensing algorithm with receiver operating characteristics (ROC) using monte carlo simulations. The ROC is a curve between Probability of detection (P_d) versus Probability of False alarm (P_f). These curves are obtained for different values of observations (N), censoring ratio (p) and SNR. We have also presented ROC for ED, AD and OS sensing algorithms and compared them with the proposed one.

Fig. 2 shows ROC for CAD sensing at SNR of -2 dB for $P_f = 0.05$ under AWGN and quasi-static rayleigh fading channel. We have taken R = 24 corresponding to the value of $N = 40$, so that p remains constant as 0.6. We can see that CAD gives higher detection performance under AWGN compared to quasi-static rayleigh fading channel as expected.

Fig. 3 shows ROC for CAD sensing for $P_f = 0.05$ and an SNR of 6 dB for fixed value of N = 25. We have taken R = 24 corresponding to the value of N, so that p remains constant as 0.6. We have also presented ROC of ED and AD sensing algorithms for the same values of P_f, N and SNR. For specified P_f and N, it can be seen that the P_d = 0.8419 is obtained in AD sensing. For CAD sensing, N = 40 and R = 24 are taken. It means 24 observations are used, which are almost same as $N = 25$ in case of AD sensing. It can be seen that better $P_d = 0.8615$ for CAD sensing is achieved compared to AD sensing. In case of ED and OS sensing, $P_d = 0.7274$, $P_d = 0.4216$ are achieved respectively. Thus, the proposed CAD sensing outperforms OS, ED and AD sensing algorithms.

Furthermore, in fig. 4, we have shown P_d versus SNR for $P_f = 0.05$, $N = 40$ and $p = 0.6$ for CAD sensing. As SNR increases, P_d increases as per expectation. We have also presented performance of ED, AD and OS sensing methods in the same figure with $N = 25$ and same P_f. We can see that $P_d = 0.1106,\ 0.1967,\ 0.3655$ and 0.4225 for ED, OS, AD and CAD respectively at SNR of -8 dB.

It can be shown from fig. 4 that when the signal is transmitted on quasi-static rayleigh fading channel, CAD sensing has almost 6 dB gain over ED sensing i.e signal detected by ED sensing at 10 dB SNR with $Pd = 0.8183$, the similar detection performance is achieved at 4 dB in case of CAD sensing at the same value of P_f. Furthermore, gain of 1 dB is archived compared to AD sensing. We can see significant improvement in P_d compared to OS sensing. Thus, the CAD sensing outperforms at lower value of SNR compared to the remaining schemes for the whole range of SNR from -10 dB to 10 dB.

IV. CONCLUSION

In this paper, we have discussed the problem of spectrum sensing as null hypothesis testing problem for censored observations under quasi-static channel. The ROC is presented for the proposed CAD algorithm and compared with conventional OS, AD and ED sensing methods. The CAD sensing method gives significant improvement in detection of primary user compared to the ED sensing at about 6 dB gain at lower signal to noise ratio. The simulation and numerical results show that CAD sensing outperforms OS sensing as well as AD sensing with a gain of 1 dB.

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